Solving systems of equations in two variables using a substitution method

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \qquad (1)$$

Using a method of substitution, for the equations of the system (1) we will substitute the unknown in the other equation of the system

Example 1. Solve the system of equations (where one of the coefficients is 1)

$$\begin{cases} 2x + y = 5 \\ 3x - 2y = 4 \end{cases}$$

From the first equations we present the unknown y through x

$$\begin{cases} y = 5 - 2x \\ 3x - 2y = 4 \end{cases}$$

The following $5 - 2x$ will be substituted with y

$$\begin{cases} y = 5 - 2x \\ 3x - 2(5 - 2x) = 4 \end{cases}$$

We solve the second equation which is now a linear equation with one unknown

$$\begin{cases} y = 5 - 2x \\ 3x - 10 + 4x = 4 \end{cases}$$

$$\begin{cases} y = 5 - 2x \\ 3x + 4x = 4 + 10 \end{cases}$$

$$\begin{cases} y = 5 - 2x \\ 7x = 14 \end{cases}$$

$$\begin{cases} y = 5 - 2x \\ x = \frac{14}{7} \end{cases}$$

$$\begin{cases} y = 5 - 2x \\ x = 2 \end{cases}$$

Once we get a value for x , we substitute that value in the first equation

$$\begin{cases} y = 5 - 2 \cdot 2 \\ x = 2 \end{cases}$$

$$\begin{cases} y = 5 - 4 \\ x = 2 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 2 \end{cases}$$

Solution to the system of equation is: $(x, y) = (2,1)$

Example 2. Solve the system of eqations (where one of the coefficients is -1)

$$\begin{cases} 3x - 4y = -6 \\ -x + 2y = 4 \end{cases}$$

From the first equations we present the unknown y through the unknown x

$$\begin{cases} 3x - 4y = -6 \\ -x = 4 - 2y \end{cases}$$

We multiple the second equation with -1

$$\begin{cases} 3x - 4y = -6 \\ x = -4 + 2y \end{cases}$$

The following $-4 + 2y$ is substituted in the first equation with x

$$\begin{cases} 3(-4+2y)-4y=-6 \\ x=-4+2y \end{cases}$$

We solve the first equation, which now represents a linear equation with one unknown

$$\begin{cases} -12+6y-4y=-6 \\ x=-4+2y \end{cases}$$

$$\begin{cases} 6y-4y=-6+12 \\ x=-4+2y \end{cases}$$

$$\begin{cases} 2y=6 \\ x=-4+2y \end{cases}$$

$$\begin{cases} y=\frac{6}{2} \\ x=-4+2y \end{cases}$$

$$\begin{cases} y=3 \\ x=-4+2y \end{cases}$$

Once we get a value for y, we substitute that value in the first equation

$$\begin{cases} y=3 \\ x=-4+2\cdot 3 \end{cases}$$

$$\begin{cases} y=3 \\ x=-4+6 \end{cases}$$

$$\begin{cases} y=3 \\ x=2 \end{cases}$$

Solution to the system of equation is: $(x,y)=(2,3)$

Example 3. Solve the system of equations (where there isn't a coefficient of 1)

$$\begin{cases} 2x+3y=5 \\ -3x+5y=2 \end{cases}$$

From the first equations we present the unknown x through y

$$\begin{cases} 2x=5-3y \\ -3x+5y=2 \end{cases}$$

$$\begin{cases} x=\dfrac{5-3y}{2} \\ -3x+5y=2 \end{cases}$$

The following $\frac{5-3y}{2}$ is substituted in the first equation with x

$$\begin{cases} x=\dfrac{5-3y}{2} \\ -3\cdot\dfrac{5-3y}{2}+5y=2 \end{cases}$$

We solve the first equation, which now represents a linear equation with one unknown

$$\begin{cases} x=\dfrac{5-3y}{2} \\ \dfrac{-15+9y}{2}+5y=2 \end{cases}$$

We multiply the second equation with 2 (to get rid of the fraction)

$$\begin{cases} x=\dfrac{5-3y}{2} \\ 2\cdot\dfrac{-15+9y}{2}+2\cdot 5y=2\cdot 2 \end{cases}$$

$$\begin{cases} x=\dfrac{5-3y}{2} \\ -15+9y+10y=4 \end{cases}$$

$$\begin{cases} x = \dfrac{5-3y}{2} \\ 9y + 10y = 4 + 15 \end{cases}$$

$$\begin{cases} x = \dfrac{5-3y}{2} \\ 19y = 19 \end{cases}$$

$$\begin{cases} x = \dfrac{5-3y}{2} \\ y = \dfrac{19}{19} \end{cases}$$

$$\begin{cases} x = \dfrac{5-3y}{2} \\ y = 1 \end{cases}$$

Once we get a value for y, we substitute that value in the first equation

$$\begin{cases} x = \dfrac{5-3\cdot 1}{2} \\ y = 1 \end{cases}$$

$$\begin{cases} x = \dfrac{5-3}{2} \\ y = 1 \end{cases}$$

$$\begin{cases} x = \dfrac{2}{2} \\ y = 1 \end{cases}$$

$$\begin{cases} x = 1 \\ y = 1 \end{cases}$$

Solution to the system of equation is: $(x, y) = (1,1)$